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STABILITY ANALYSIS OF THE COMPRESSIBLE, ADIABATIC, SIMILAR BOUN--ETC(U)
MAY 81 G R VERMA, S J SCHERR, W L HANKEY AFOSR-80-0150
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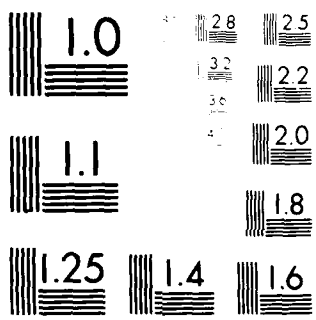

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STABILITY ANALYSIS OF THE COMPRESSIBLE, ADIABATIC,
SIMILAR BOUNDARY LAYER EQUATIONS (LOWER BRANCH)

G.R. VERMA, W.L. HANKEY, S.J. SCHERR

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In a previous report, the authors analyzed the stability of the lower branch solutions of the incompressible ($M_\infty=0$) Falkner-Skan boundary layers. This report extends this work by including the effect of Mach number M_∞ on the stability of adiabatic case.

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FOREWORD

This report is the result of work carried on in Computational Aerodynamics Group, Flight Dynamics Laboratory, Wright Patterson Air Force Base by Dr. G. R. Verma, Dr. W. L. Hankey and Mr. S. J. Scherr, from June 15, 1980 to August 20, 1980. During this period Dr. Verma's work was supported by a grant from Air Force Office of Scientific Research (Grant # AFOSR80-0150). Additional support was provided under project 2307N436. The authors would like to thank the Air Force Systems Command, Air Force Office of Scientific Research and Wright Patterson Air Force Base for providing resources for the senior author to spend the summer of 1980 at WPAFB.

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SECTION I

INTRODUCTION

In [2] the authors analyzed the stability of the lower branch solutions of the incompressible ($M_\infty = 0$) Falker-Skan boundary layers. There a perturbation analysis to these boundary layers was performed resulting in the Rayleigh stability equation. Eigen value solutions were obtained for the Rayleigh equation for different adverse pressure gradient (β) values. All retarded flows were found to be unstable for a small range of frequencies with the amplification factor increasing as the extent of reversed flow increased.

In the present report we have entended that work by including the effect of Mach number M_∞ on the stability of adiabatic ($S_w = 0$) Falker-Skan equations for $\beta = -.04, -.08, -.12, -.16$ and $-.19884$. We found out that in all these cases as the Mach number M_∞ increases the instability of flow decreases. In most of the cases the instability almost completely disappeared at $M_\infty = 3$.

SECTION II

GOVERNING EQUATIONS

In this report the stability of compressible inviscid separated flows will be anlayzed.

Two dimensional Navier-Stokes equations are as follows

$$U_t + E_x + F_y = 0 \tag{2.1}$$

where the vectors U, E, and F are given as

$$U = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho e \end{pmatrix}, \qquad E = \begin{pmatrix} \rho u \\ \rho u^2 - \sigma_{11} \\ \rho uv - \tau_{12} \\ \rho ue - (\underline{V} \cdot \tau)_x - \dot{q}_x \end{pmatrix}$$

$$F = \begin{pmatrix} \rho u \\ \rho uv - \tau_{12} \\ \rho u \ - \sigma_{22} \\ \rho ve - (\underline{V} \cdot \tau)_y - \dot{q}_y \end{pmatrix} \tag{2.2}$$

where

$$\sigma_{11} = -p + \lambda \nabla \cdot \underline{V} + 2\mu u_x$$

$$\tau_{21} = \tau_{12} = \mu(u_y + v_x)$$

$$\sigma_{22} = -p + \lambda \nabla \cdot \underline{V} + 2\mu v_y$$

$$\dot{q}_x = KT_x \ ; \ \dot{q}_y = KT_y$$

$$\underline{V} = \begin{pmatrix} u \\ v \end{pmatrix}$$

and $$\underline{\underline{\tau}} = \begin{pmatrix} \sigma_{11} & \tau_{12} \\ \tau_{21} & \sigma_{22} \end{pmatrix} \tag{2.3}$$

Two approximations to these equations are required i.e. Steady boundary layer approximations and unsteady inviscid equations.

1. For steady boundary layer approximation

$$\underline{V}_t = 0$$

$$\sigma_{11} \doteq - p$$

$$\tau_{12} = \tau_{12} = \mu\, u_y \equiv \tau$$

$$\sigma_{22} = -p$$

$$\dot{q}_x = 0 \ ; \ \dot{q}_y = KT_y \tag{2.4}$$

$$(\underline{V} \cdot \underline{\underline{\tau}})_x = -up$$

$$(\underline{V} \cdot \underline{\underline{\tau}})_y = -vp + u\tau$$

and the enthalpy $H = e + \frac{p}{\rho} = C_v T + RT + \frac{u^2}{2} = C_p T + \frac{u^2}{2}$

Hence the Navier-Stokes equations reduce to

$$(\rho u)_x + (\rho v)_y = 0 \tag{2.5}$$

$$(\rho u^2 + p)_x + (\rho uv - \tau)_y = 0 \tag{2.6}$$

$$p_y \approx 0 \tag{2.7}$$

$$(\rho uH)_x + (\rho vH - \mu H_y)_y = 0 \tag{2.8}$$

2. For unsteady inviscid equations

$$\sigma_{11} = \sigma_{22} = -p$$

$$\tau_{12} = \tau_{12} = 0$$

$$\dot{q}_x = \dot{q}_y = 0 \tag{2.9}$$

$$(\underline{V} \cdot \underline{\underline{\tau}})_x = -up \ ; \ (\underline{V} \cdot \underline{\underline{\tau}})_y = -vp$$

and $H = C_p T + \frac{u^2 + v^2}{2}$

Hence u is the same as in equation (2.2) and

$$E = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ \rho uH \end{pmatrix} ; \qquad F = \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ \rho vH \end{pmatrix} \tag{2.10}$$

These two sets of equations are used to find two different solutions. The boundary layer equations are solved for similar flows and the unsteady inviscid equations are used to examine perturbations of the steady boundary layer flow to investigate stability.

Transforming the equations (2.5), (2.6) and (2.8) by the following transformations [1]

$$d\xi = \frac{p_e a_e}{p_o a_o}\, dx \tag{2.11}$$

$$d\eta = N \frac{a_e \rho}{a_o \rho_o}\, dy; \; N = \left(\frac{m+1}{2} \frac{a_o}{\nu_o} \frac{Me}{\xi}\right)^{1/2} \tag{2.12}$$

$$p_x = -\rho_e\, u_e\, u_{ex} \tag{2.13}$$

and letting

$$f'(\eta) = \frac{u}{ue} \tag{2.14}$$

$$\frac{H(\eta)}{H_e} = 1 + S \tag{2.15}$$

$$M_e = C\xi^m \tag{2.16}$$

we obtain the following equations

$$f''' + ff'' = \beta\,(f'^2 - S - 1) \tag{2.17}$$

$$S'' + f\,S' = 0 \tag{2.18}$$

Now let us assume small perturbations of the form

$$u = \bar{u}\,(y) + u'\,(x, y, t) \tag{2.19}$$

$$v = 0 + v'\,(x, y, t) \tag{2.20}$$

$$p = p_\infty + p'\,(x, y, t) \tag{2.21}$$

$$\rho = \bar{\rho}\,(y) + \rho'\,(x, y, t) \tag{2.22}$$

Substituting these into equations of continuity, momentum and energy; and retaining the terms of first order only we obtain

$$\frac{\rho'_t}{\bar\rho} + \bar u \frac{\rho'_x}{\bar\rho} + \frac{\bar\rho_y}{\bar\rho} v' + u'_x + v'_y = 0 \tag{2.23}$$

$$u'_t + \bar u\, u'_x + \bar u_y\, v' + \frac{p'_x}{\bar\rho} = 0 \tag{2.24}$$

$$v'_t + \bar u\, v'_x + \frac{p'_y}{\bar\rho} = 0 \tag{2.25}$$

$$\bar T\, C_p \left[\frac{1}{p_\infty}\left(\frac{\partial p'}{\partial t} + \bar u \frac{\partial p'}{\partial x}\right) - \frac{1}{\bar\rho}\left(\frac{\partial p'}{\partial t} + \bar u \frac{\partial p'}{\partial x}\right)\right] + \bar H_y\, v' + \bar u \left(\frac{\partial u'}{\partial t} + \bar u \frac{\partial u'}{\partial x}\right) = \frac{p'_t}{\bar\rho} \tag{2.26}$$

By eliminating ρ' from (2.23) and (2.26) we obtain

$$\frac{1}{p_\infty}\left(\frac{\partial p'}{\partial t} + \bar u \frac{\partial p'}{\partial x}\right) + \gamma(u'_x + v'_y) = 0 \tag{2.27}$$

Let $u' = \hat u\,(y)\, e^{i\alpha(x-ct)}$ (2.28)

$v' = \hat v\,(y)\, e^{i\alpha(x-ct)}$ (2.29)

$p' = \hat p\,(y)\, e^{i\alpha(x-ct)}$ (2.30)

Then the equations (2.24), (2.25), and (2.27) reduce to

$$i\alpha(\bar u - c)\,\hat u + \bar u_y\,\hat v + i\alpha\frac{\hat p}{\bar\rho} = 0 \tag{2.31}$$

$$i\alpha(\bar u - c)\,\hat v + \frac{\hat p_y}{\bar\rho} = 0 \tag{2.32}$$

$$i\alpha(\bar u - c)\,\frac{\hat p}{\bar\rho} + \frac{\gamma p_\infty}{\bar\rho}(\hat v_y + i\alpha\hat u) = 0 \tag{2.33}$$

Let $\Phi = \hat v$, eliminate $\hat u$ and $\hat p$ from the equations (2.31), (2.32) and (2.33) and use the relation $\bar\rho = p_\infty/R\bar T$, and we get

$$\left[\frac{(\bar u - c)\,\Phi_y - \bar u_y \Phi}{\gamma R\bar T - (\bar u - c)^2}\right]_y = \frac{\alpha^2(\bar u - c)\Phi}{\gamma R\bar T} \tag{2.34}$$

Now from (2.15) we can obtain a relationship for $\gamma R\bar{T}$ as a function of $S(\eta)$ and $\bar{u}$.

$$\gamma R\bar{T} = \frac{1+S - 0.2\, M_o^2\, \bar{u}^2}{M_o^2} \tag{2.35}$$

where

$$M_o^2 = \frac{M_\infty^2}{1+0.2\, M_\infty^2} \tag{2.36}$$

Using 2.14 the equation (2-34) becomes

$$\left[\frac{(f' - \bar{c})\Phi_\eta - f''\Phi}{gh}\right]_\eta = \frac{\alpha^2 h(f' - \bar{c})\,\Phi}{1+S-0.2\, M_o^2\, f'^2} \tag{2.37}$$

where

$$g = 1 + S - 0.2\, M_o^2\, f'^2 - (f' - \bar{c})^2 \tag{2.38}$$

$$h = \frac{d\eta}{dy} = \frac{\Psi^4}{N}\left[1 - \frac{\Psi-1}{\Psi}\, f'^2\right] \tag{2.39}$$

$$\Psi = 1 + 0.2\, M_\infty^2 \tag{2.40}$$

and $$N = \left(\frac{m+1}{2}\, \frac{\alpha_o}{\nu_o}\, \frac{Me}{\xi}\right)^{1/2} \tag{2.41}$$

we solve numerically the eigen value problem given by the equation (2.37) for $S = 0$ and the boundary conditions $\Phi(0) = 0$, and $\Phi(\infty) = 0$. f' and f'' are obtained from equation (2.17) with boundary conditions $f(0) = 0$, $f'(0) = 0$ and $f'(\infty) = 1$.

SECTION III

NUMERICAL PROCEDURE

Eigen values were determined by a shooting method[2]: starting with a given boundary condition, integrating over the range of η and comparing the result with the outer boundary condition, namely $\Phi = 0$ at η_{max}. The process involved minimization of the error in the outer boundary condition which was chosen to be the square of the norm of Φ, $|\Phi|^2 = \Phi_r^2 + \Phi_i^2$. The integration was done using a fourth order Runge-Kutta method.

The method of finding eigen values utilized the same minimization routine as in [2]. Starting from a given guess the routine searched along a constant line of C_i with increasing steps until it found a relative minimum of the error. It then used the last three calculated points to determine a parabola, with the C_r value at the vertex used as the latest approximation. Then this value of C_r was held constant and a search along a line of changing C_i was carried out. After a new minimum was found, the quadratic approximation was again used to determine a new value for C_i. The third step involved searching the line connecting the original guess and the new point. After finding a minimum and utilizing quadratic approximation, the error was checked to see if it was less than some preset limit. If not, the routine started again with the latest value used in place of the original guess.

SECTION IV

RESULTS

Several cases were computed for β values of -.0001, -.04, -.08, -.12, -.16 and -.19884; at Mach numbers M_∞ of 0, 1, 2, 3 and 4. For a wide range of α values the eigen values were ascertained. These values are tabulated on the following pages (Tables 1-a to 6-d).

The values of C_i versus α are plotted for the above mentioned values of β and that of M_∞ (Figures 1-7). The values of C_i versus C_r are also plotted for all the above mentioned values of β and for $M_\infty = 0, 1$ (Figures 8-12).

Solutions were obtained with convergence error criteria of at least 10^{-6} for all cases.

The solutions obtained for the Rayleigh instability in this report are restricted to subsonic disturbances, $C_r > 1 - M_\infty^{-1}$, that die off exponentially [3]. Instabilities for $C_r < 1-M_\infty^{-1}$ are supersonic disturbances. Such solutions represent sound waves associated with the boundary layer and are not considered here.

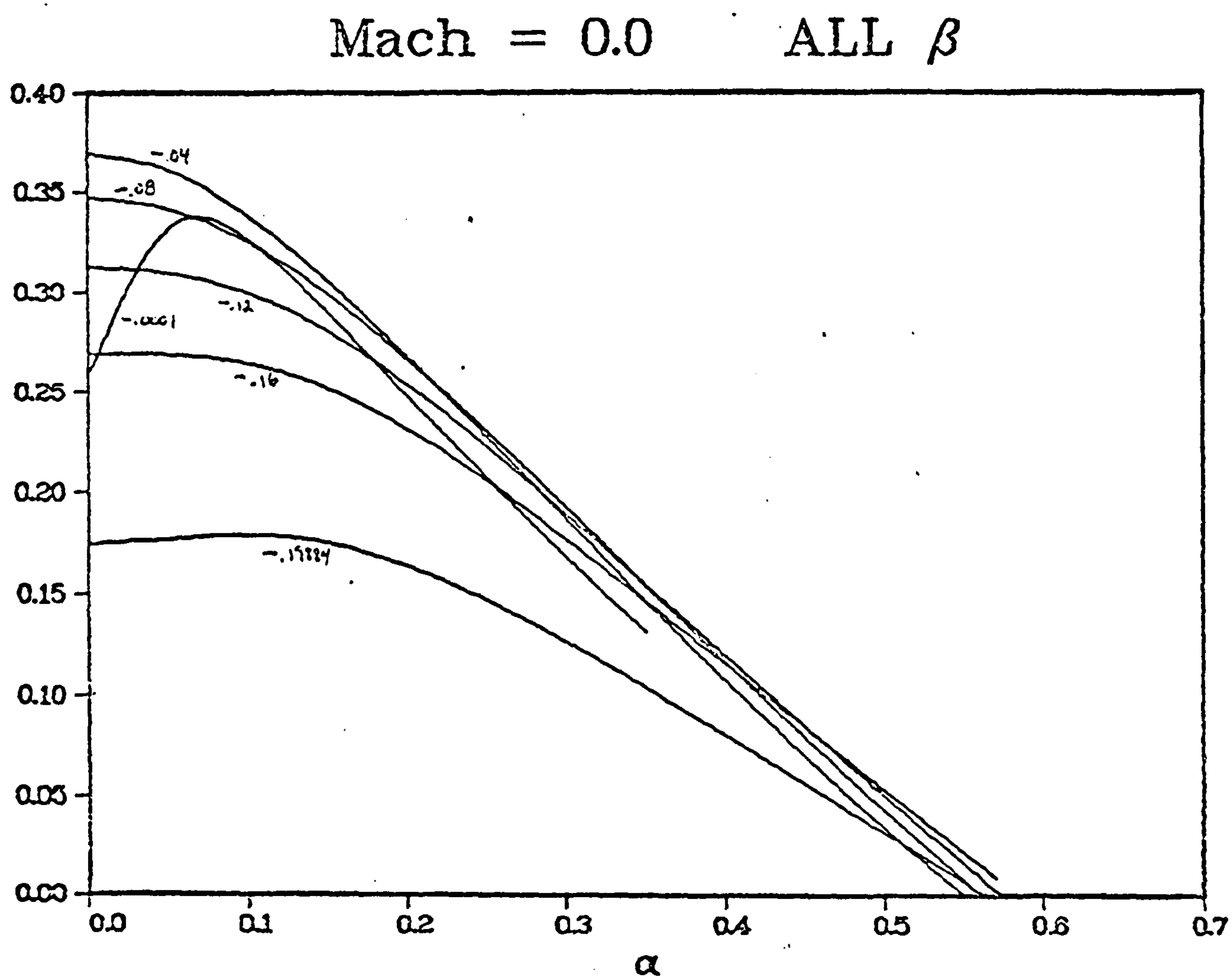

Fig 1 C_i versus α for $M_\infty = 0$ and for all values of β

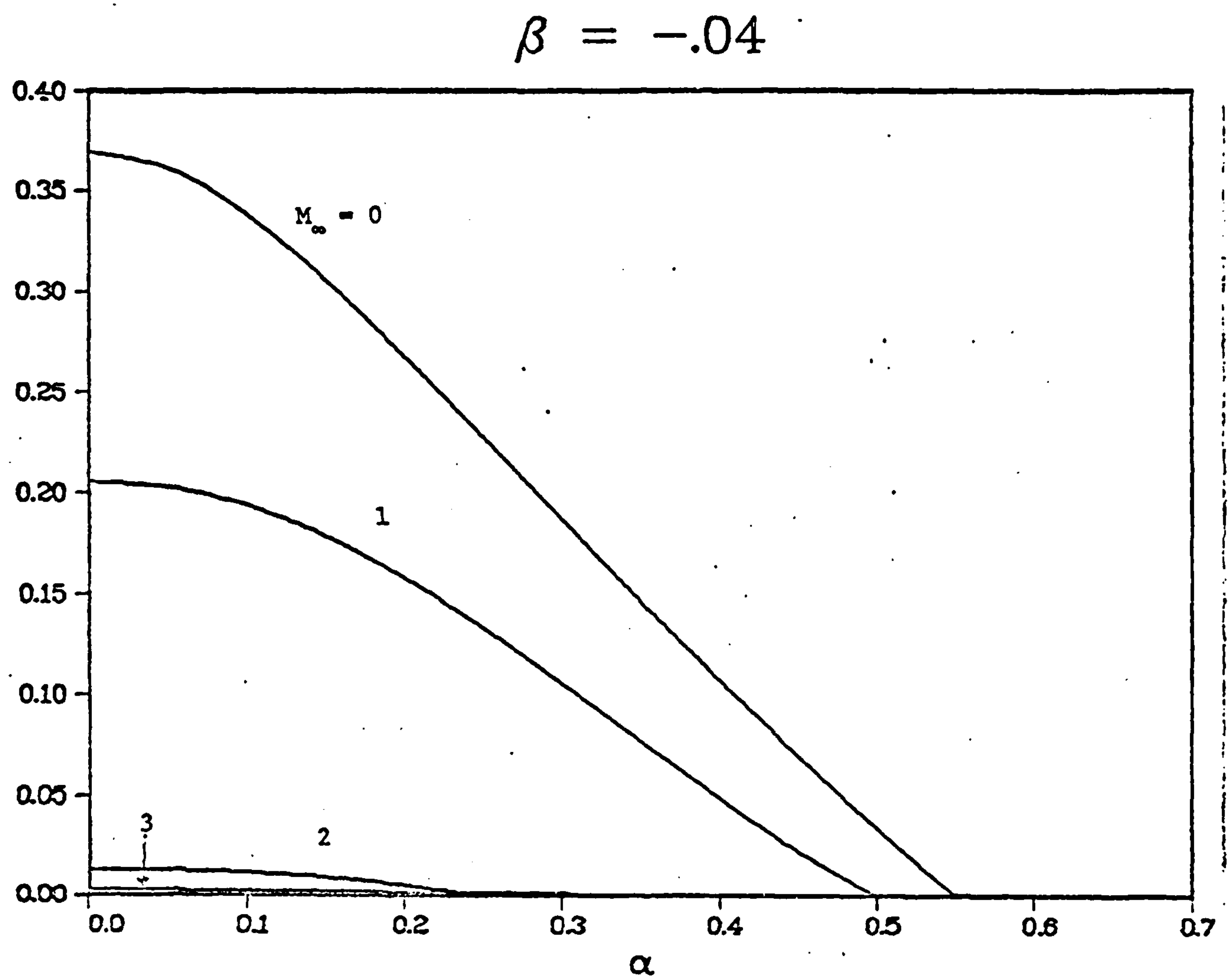

Fig 2 C_l versus α for $\beta = -.04$ and $M_\infty = 0, 1, 2, 3$

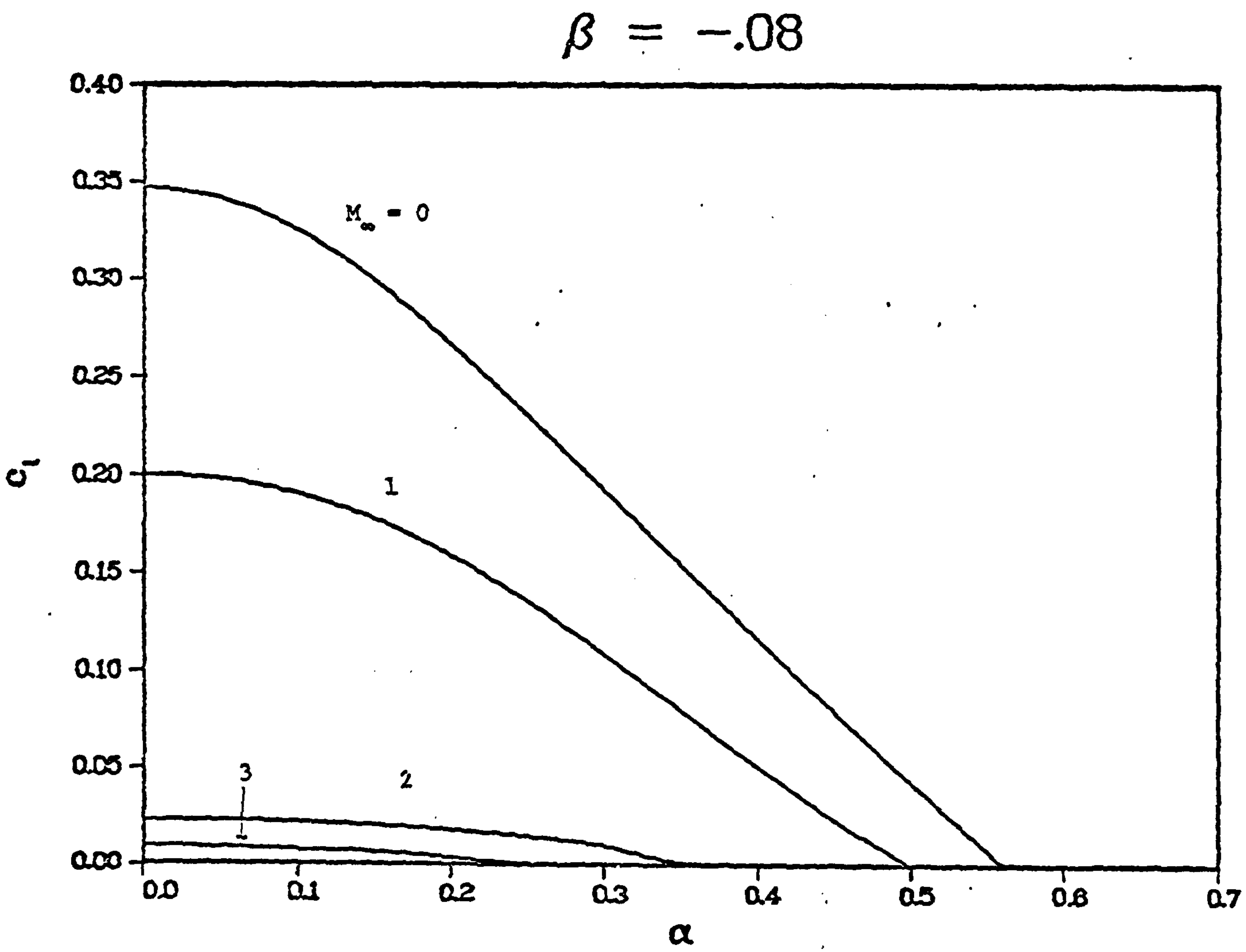

Fig 3 C_l versus α for $\beta = -.08$ and $M_\infty = 0, 1, 2, 3$

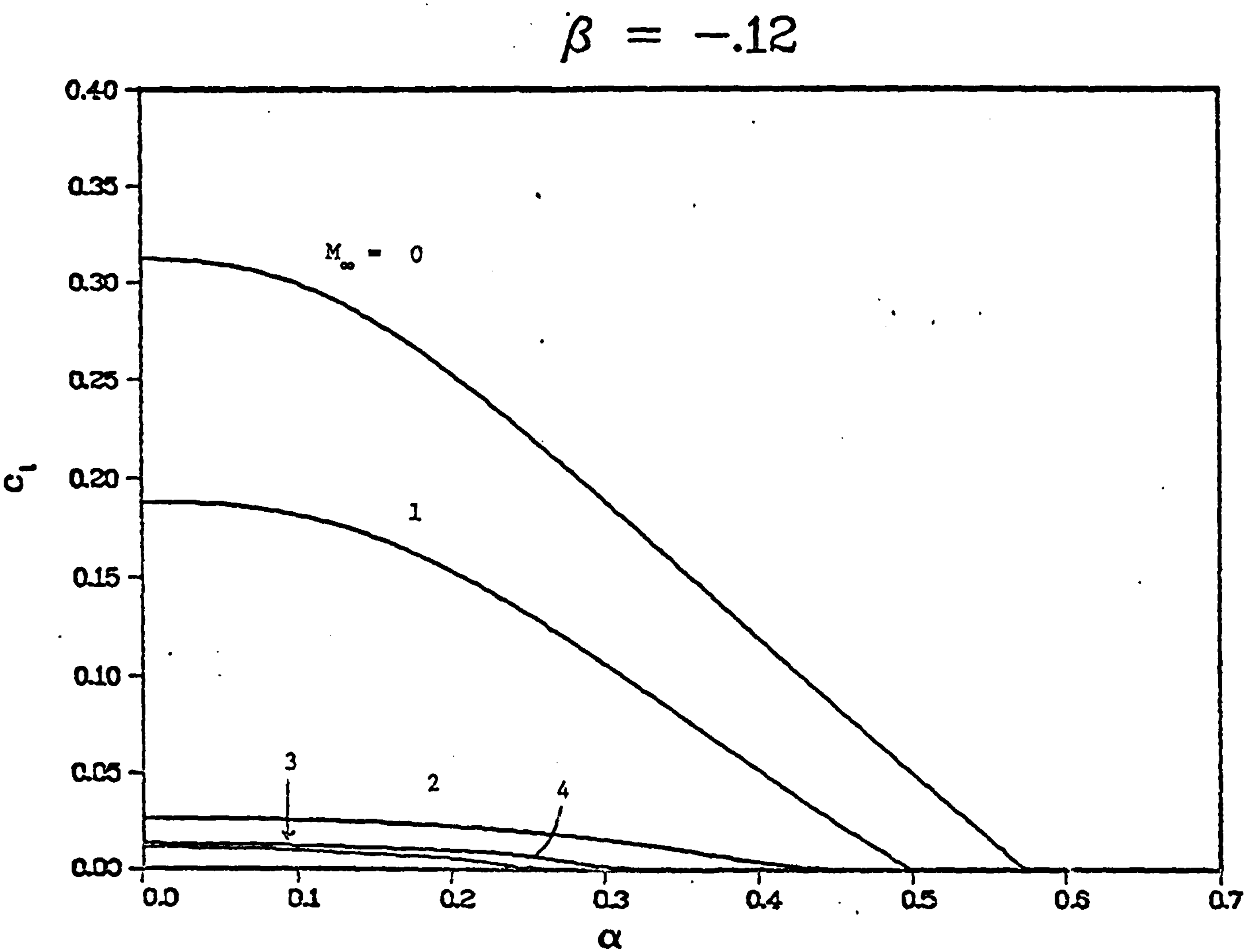

Fig 4 C_l versus α for $\beta = -.12$ and $M_\infty = 0, 1, 2, 3, 4$

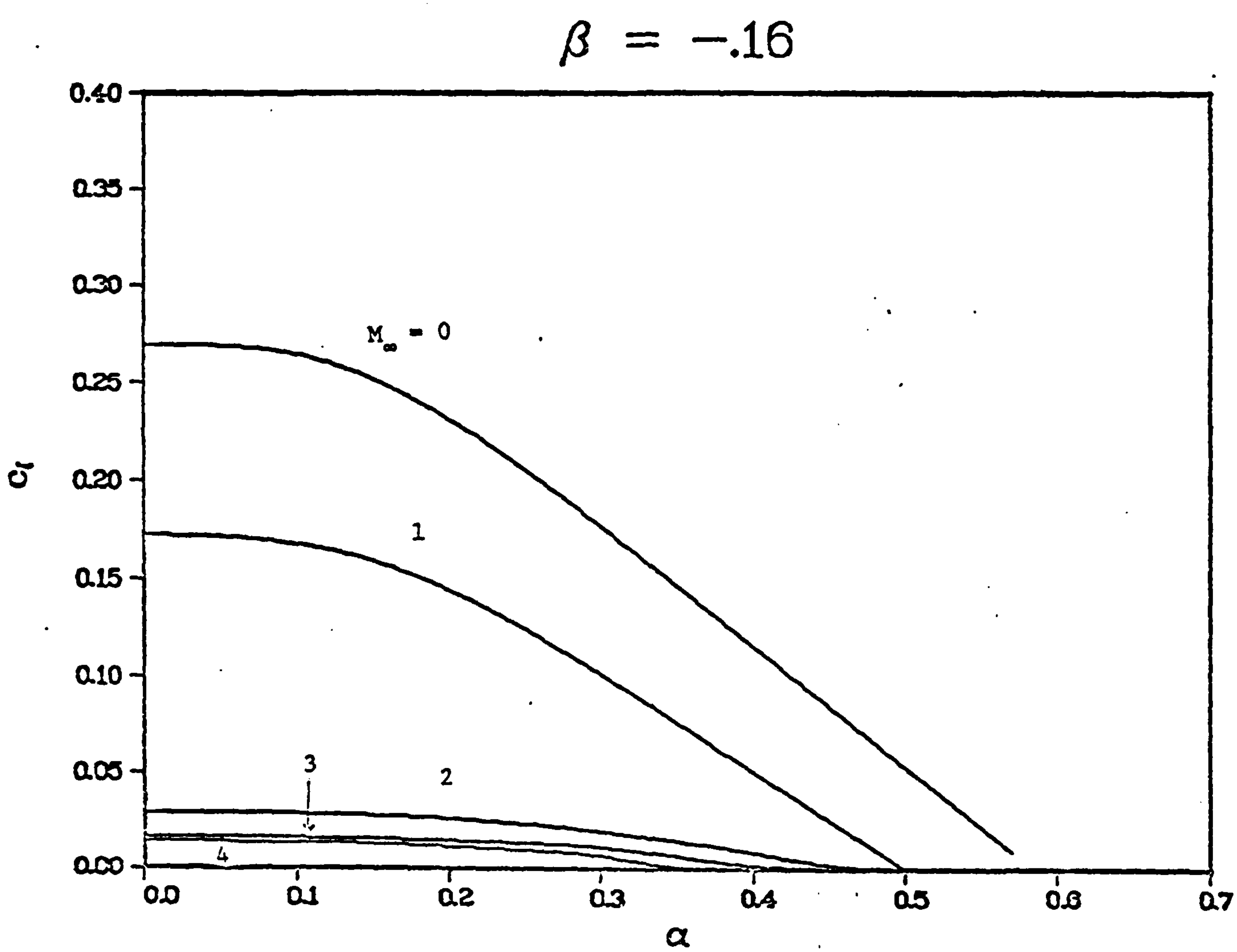

Fig 5 C_i versus α for $\beta = -.16$ and $M_\infty = 0, 1, 2, 3, 4$

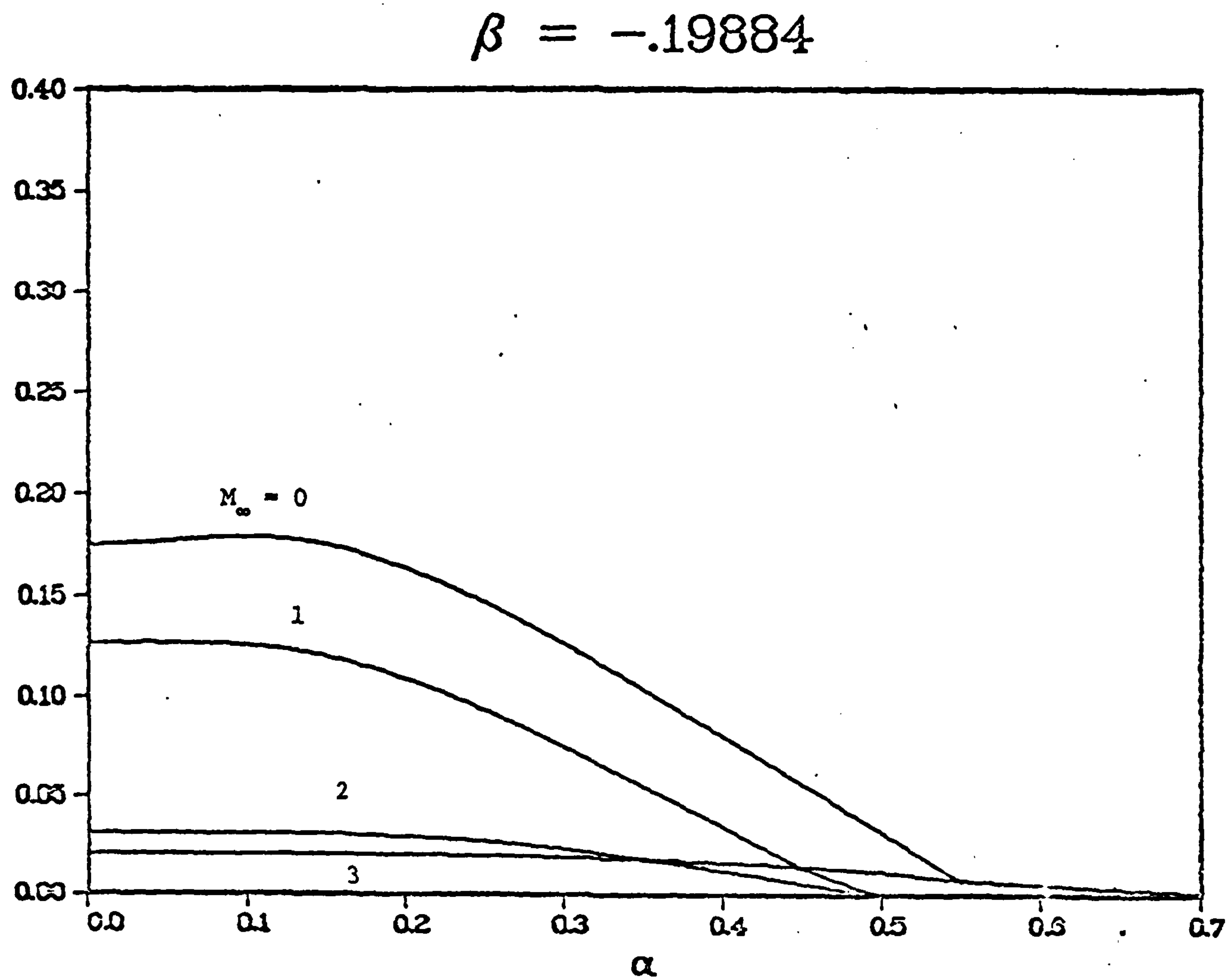

Fig 6 C_i versus α for $\beta = -.19884$ and $M_\infty = 0, 1, 2, 3$

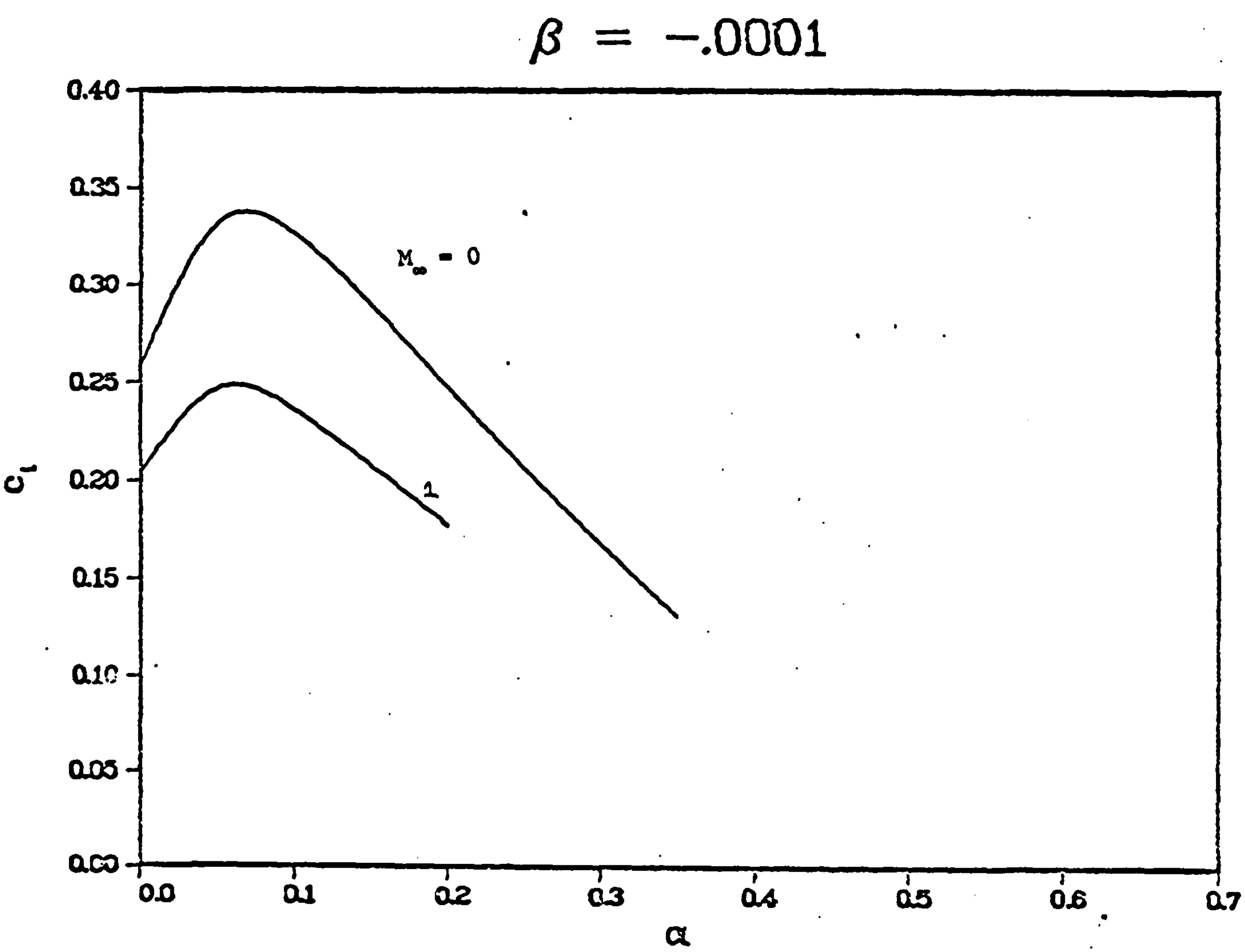

Fig 7 C_l versus α for $\beta = -.0001$ and $M_\infty = 0, 1$

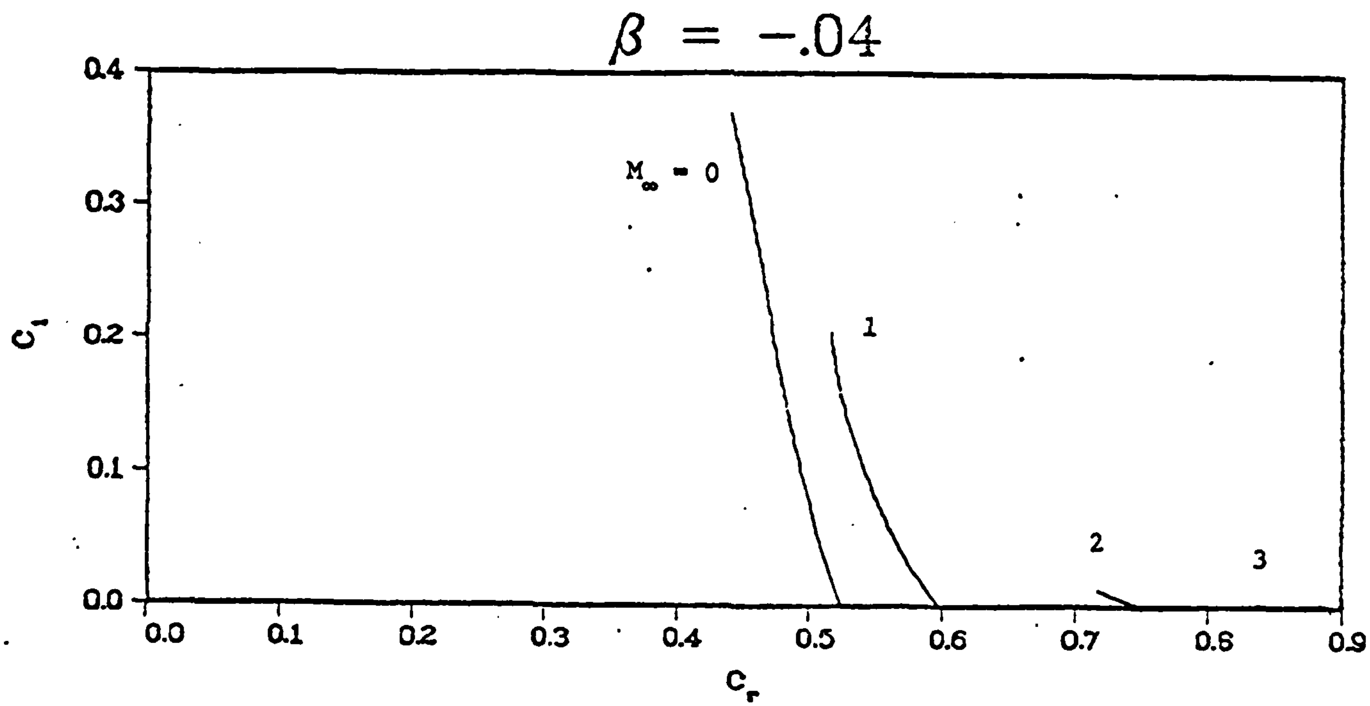

Fig 8 C_i versus C_r for $\beta = -.04$ and $M_\infty = 0, 1$

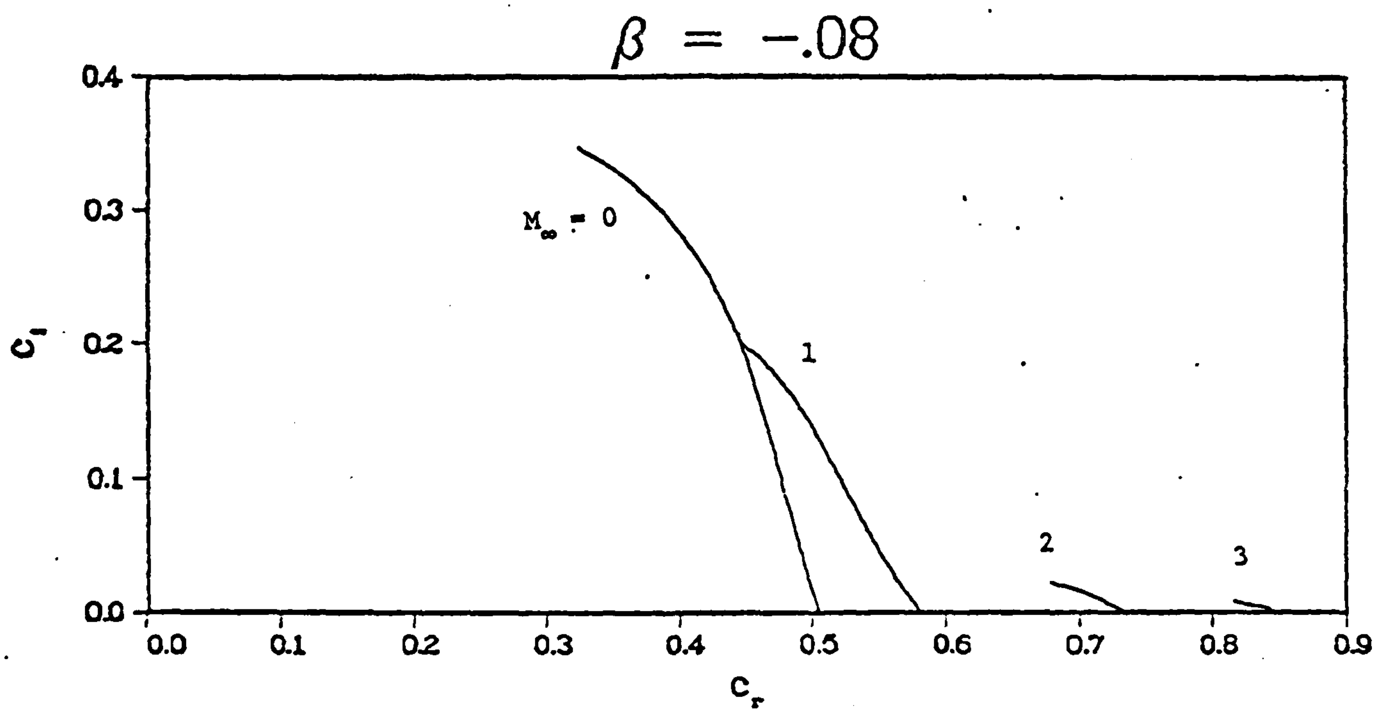

Fig 9 C_i versus C_r for $\beta = -.08$ and $M_\infty = 0, 1, 2, 3$

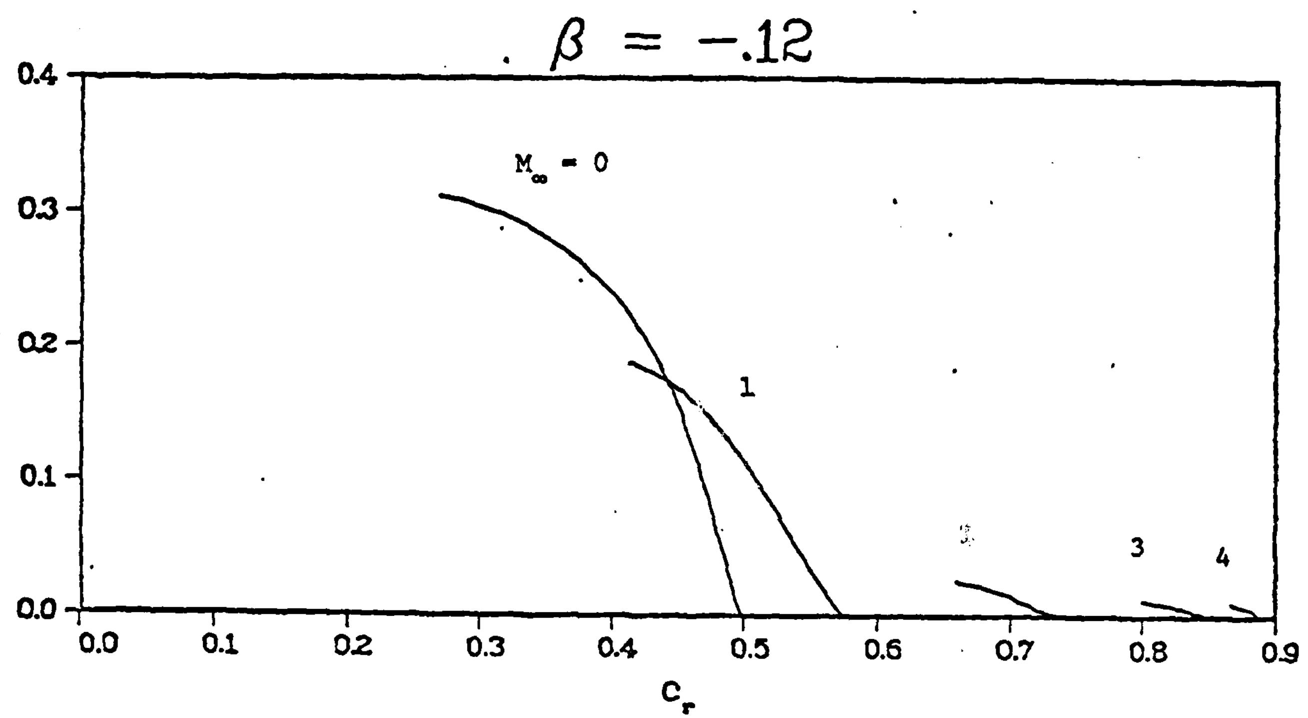

Fig 10 C_i versus C_r for $\beta = -.12$ and $M_\infty = 0, 1, 2, 3, 4$

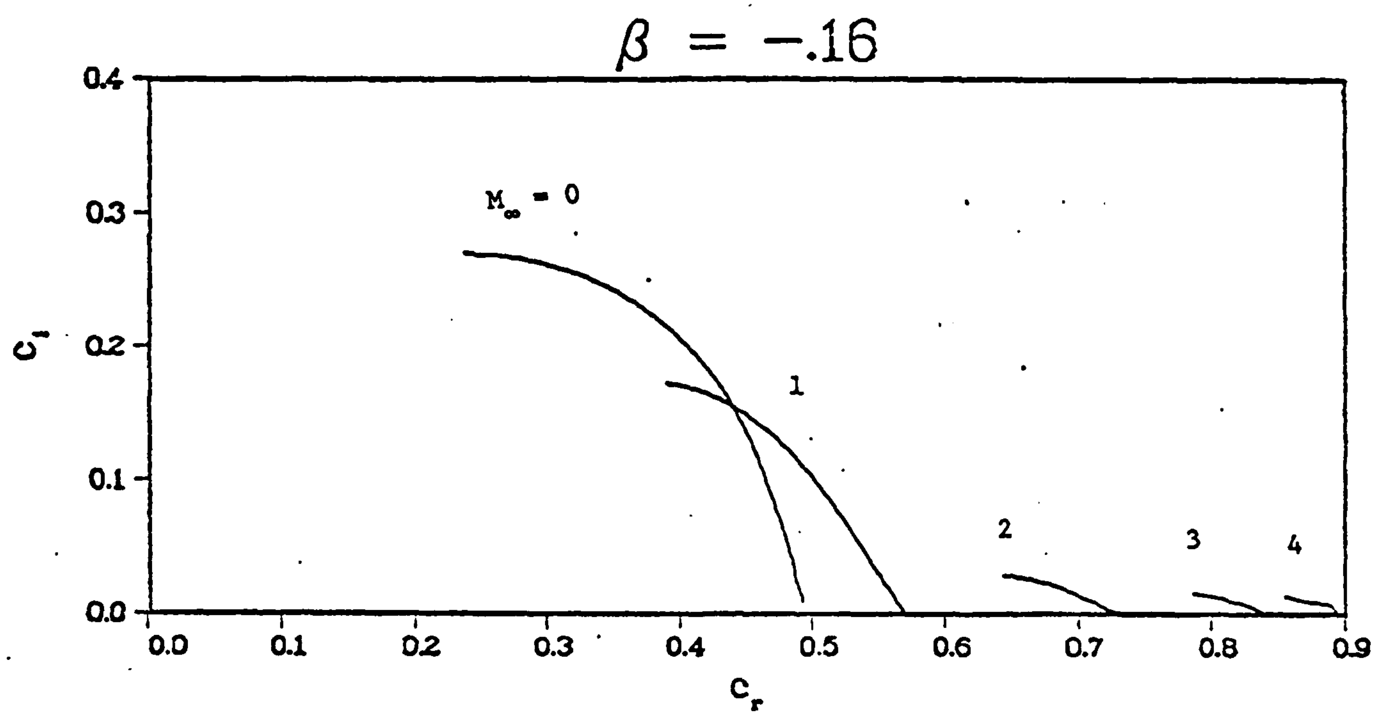

Fig 11 C_i versus C_r for $\beta = -.16$ and $M_\infty = 0, 1, 2, 3, 4$

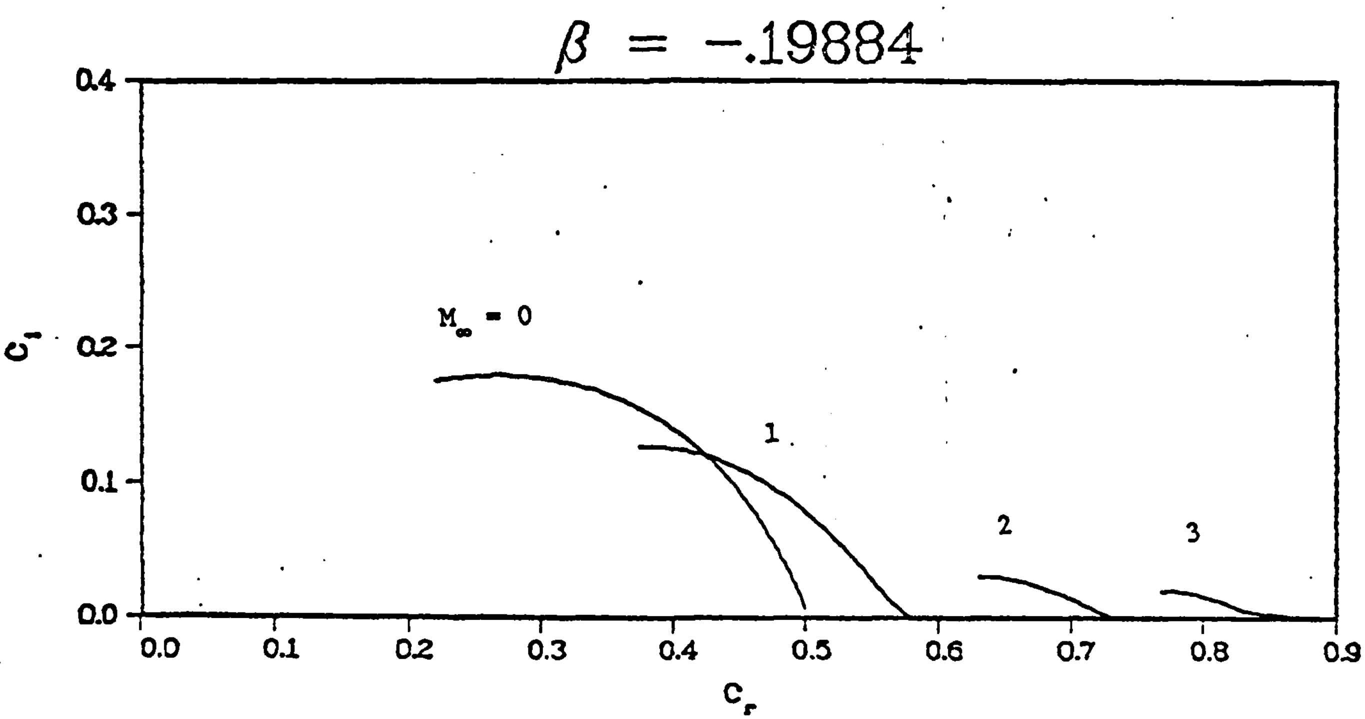

Fig 12 C_i versus C_r for $\beta = -.19884$ and $M_\infty = 0, 1\ \ 2\ \ 3$

SECTION V

SUMMARY

The stability of adiabatic compressible similar boundary layer flows (Stewartson's lower branch) has been investigated by utilizing the linearized equations resulting from a small perturbation analysis. These flows are representative of a wide class of separated boundary layers with distinct inflection points. Eigen value solutions for this equation were obtained for β = -.0001, -.04, -.08, -.12, -.16, and -.19884; for Mach number M_∞ = 0, 1, 2, 3 and 4; and for a wide range of values of α. In all these cases we found that as the Mach number increases the instability of the flow decreases. In most of the cases the instability almost completely disappeared at $M_\infty = 3$.

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2. Verma, G. R., Hankey, W. L., and Scherr, S. J., "Stability Analysis of the Lower Branch Solutions of the Falkner-Skan Equations", AFFDL-TR-79-3116, July 1979.

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EIGEN VALUES FROM STABILITY ANALYSIS FOR

ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 1-a

$\beta = -.0001$ $M_\infty = 0.0$

α	CREAL	CIM
.00	.878	.258
.05	.74227622339366	.33117005357051
.10	.62883283474993	.32643813990696
.15	.5760866669002	.28969457316972
.20	.55340765412227	.247903515584
.25	.54570288639066	.20695656175169
.30	.54565311752934	.16808272454592
.35	.5496397866896	.13139818720763

EIGEN VALUES FROM STABILITY ANALYSIS FOR

ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 1-b

$\beta = .0001$ $M_\infty = 1.0$

α	CREAL	CIM
.00	.802	.204
.05	.73081759775908	.24695656548369
.10	.65039499400247	.2363699683213
.15	.6133256194012	.20817418970878
.20	.5972291116267	.17769992745518

EIGEN VALUES FROM STABILITY ANALYSIS FOR

ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 2-a

$\beta = -.04$ $\qquad\qquad$ $M_\infty = 0.0$

α	CREAL	CIM
0.0	.43987767455304	.36885631925121
.05	.44207033341602	.3606363192512
.10	.44724820878426	.3378303798829
.15	.45401920298779	.30524424618984
.20	.46108000640763	.2675051782972
.25	.46817052480382	.22745276163212
.30	.47551265463347	.18685
.35	.48341457993034	14668814199572
.40	.49212825362202	.10759013802
.45	.50171799368214	.069965689915
.47	.50580067135532	.055371705695
.49	.51001806425015	.041065487075231
.51	.51436529442526	.027030324511536
.53	.51883654167218	.013274339319087
.54	.52112824239891	.0064672192105841
.55	.52343768083266	$.13704612334(10)^{-10}$

EIGEN VALUES FROM STABILITY ANALYSIS FOR
ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 2-b

$\beta = -.04$ $M_\infty = 1.0$

α	CREAL	CIM
.00	.51648881582676	.2056027813269
.05	.516763212275919	.20273
.10	.51777695111447	.19362
.15	.5200501719058	.17843699199205
.20	.52426129209281	.15782
.25	.53079481224409	.13315925675218
.30	.53991931058383	.10573513638387
.35	.55166905562652	.077245446948473
.40	.56553380221615	.049176841296
.45	.5810048195671	.022399128310204
.50	.59760335005259	$.78894609315189(10)^{-13}$

EIGEN VALUES FROM STABILITY ANALYSIS FOR

ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 2-c

$\beta = -.04$ $\qquad$ $M_\infty = 2.0$

α	CREAL	CIM
.00	.71652009701362	.012668266983099
.05	.71753501411812	.01226328176528
.10	.72066161383724	.0110228412935[illegible]
.15	.72618415638285	.00869459858322
.20	.73502485926542	.004771749891212
.25	.75196718703684	.00018535327544368
.30	.76419444077515	$.19202271368789(10)^{-7}$

EIGEN VALUES FROM STABILITY ANALYSIS FOR

ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 2-d

$\beta = -.04$ $\qquad\qquad$ $M_\infty = 3.0$

$\underline{\alpha}$	CREAL	CIM
.00	.83828497991129	.0022712861124202
.05	.83832353291369	.002203142045787
.10	.83837971374505	.0020331420457871
.15	.83846438764227	.0017653324015645
.20	.83859057338825	.0014442186050454
.25	.8387492371429	.001113135795937
.30	.83893095785374	.00078786558540471
.35	.83913890358613	.0005050186776936
.40	.83936245428606	.00027361258321498
.45	.83958450670098	.00010119992619257
.50	.83694157408348	$.19958331395(10)^{-6}$

EIGEN VALUES FROM STABILITY ANALYSIS FOR

ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 2-e

$\beta = -.04$ $M_\infty = 4.0$

$\underline{\alpha}$	CREAL	CIM
.30	.89821345486853	.000021922152712024
.35	.8986111670579	.0004611158595219
.40	.89953752407322	.0028801200643778
.45	.89510209749401	.045208127853594
.50	.87404992469655	.045208127853594
.55	.86658125716397	.031823395903264
.60	.87048122749137	.022711831051369
.65	.87609006759250	.017925665965007
.70	.88688808123776	.014655665965008
.75	.88728460663517	.031233426575069
.85	.8762373558128	.021616774127454
.90	.87964029961504	.016694475973842
.95	.8838[illegible]11569359	.01370968395625
1.00	.89268429567826	.012694502651197
1.05	.88972705279445	.021556311009098
1.10	.88499035977476	.017742086117608
1.15	.88669858993286	.01254227480618
1.20	.88971509780423	.0097196462037101
1.25	.89321141504163	.0075396680289642
1.30	.90053222429137	.010810059035796

EIGEN VALUES FROM STABILITY ANALYSIS FOR

ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 2-f

$\beta = -.04$ $M_\infty = 5.0$

α	CREAL	CIM
.25	.925709228969	$.10764531469363(10)^{-7}$
.30	.92599278831718	.00052922822359162
.35	.92662183965195	.0036208198669419
.40	.92815396787939	.044768670567448
.45	.90527927877179	.043548670567449
.50	.89973634086423	.029960914691319
.55	.90339925807196	.022265546403671
.60	.91104518004842	.016848106851863
.65	.919499823277	.03291795747694
.70	.90713307942075	.034841795747696
.75	.90287171885508	.027562290719606
.80	.90488432472496	.022291710386326
.85	.90924818100037	.019063122959966
.90	.91656201320373	.024864825434707
.95	.90980435312741	.028204726049801
1.00	.90675258382852	.024018271420318
1.05	.9080509069683	.02020342004305
1.10	.91094054585563	.017880372690646
1.15	.91619267667693	.019430372690645
1.20	.91302631906102	.022852784863159
1.25	.91094622674332	.020017881963299

EIGEN VALUES FROM STABILITY ANALYSIS FOR

ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 3-a

$\beta = -.08$ $M_\infty = 0.0$

α	CREAL	CIM
0.0	.32369623377884	.34699688870065
.05	.33369584191324	.34136688870062
.10	.35826638444223	.32487687953082
.15	.3871139446555	.29876721357765
.20	.41300148863976	.26621778203783
.25	.43359629937571	.23017553362009
.30	.44940833839654	.19254904411
.35	.46187743256269	.15449844555
.40	.47244512611985	.11668512889899
.45	.48225255493466	.079432215574946
.50	.49209935812562	.043039565854083
.51	.49412146647788	.035884937258694
.52	.49616827571136	.028775493482445
.53	.49824205645293	.021712720558712
.54	.5003447834241	.014698024786192
.55	.5024780843498	.007731837782228
.56	.50464325465667	.00081805807540883
.57	.50676888027108	$.72384625813(10)^{-11}$

EIGEN VALUES FROM STABILITY ANALYSIS FOR

ADIABATIC COMPRESSIBLE REVRSED FLOW BOUNDARY LAYERS

Table 3-b

$\beta = -.08$ $\qquad\qquad$ $M_\infty = 1.0$

$\underline{\alpha}$	CREAL	CIM
0.0	.44717644074037	.20047
.05	.45036889272004	.19827
.10	.45922151788401	.19084
.15	.47191142328282	.17768036911841
.20	.48650885313897	.15878544735304
.25	.50145992619499	.13522670533432
.30	.51645114109554	.10840338464129
.35	.53169707037833	.079953633930761
.40	.54766418019842	.051166391960728
.45	.56449725740563	.023300624031718
.50	.58199860635478	.20484929$(10)^{-10}$

EIGEN VALUES FROM STABILITY ANALYSIS FOR

ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 3-c

$\beta = -.08$ $\qquad\qquad$ $M_\infty = 2.0$

$\underline{\alpha}$	CREAL	CIM
.00	.6786161709683	.022508215380253
.05	.67972441881664	.022250750848725
.10	.68303497061643	.021437835780906
.15	.68850427971181	.019955132584977
.20	.69605683760054	.017631827672087
.25	.70562456196254	.014255879594706
.30	.71733360169382	.0095062598734696
.35	.73448129706239	.00099907312154349
.40	.74207518088353	.23000236908$(10)^{-7}$

EIGEN VALUES FROM STABILITY ANALYSIS FOR

ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 3-d

$\beta = -.08$ $M_\infty = 3.0$

α	CREAL	CIM
.00	.81598597441035	.00878
.05	.81703718909398	.0083
.10	.82160858194915	.0070816257624878
.15	.82804044677527	.0061516257624876
.20	.84264511987827	.0032723347777689
.25	.84482479602233	.00024791089280703
.30	.83193057606446	$.22978827129375(10)^{-7}$

EIGEN VALUES FROM STABILITY ANALYSIS FOR

ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 4-a

$\beta = -.12$ $M_\infty = 0.0$

α	CREAL	CIM
0.0	.26797687730572	.31234232980631
0.5	.28102153388006	.30951232980998
.10	.31347137430098	.29945827726145
.15	.35178357956826	.2803755997
.20	.3862991794655	.25380503131086
.25	.41390661598813	.22262760308578
.30	.43484237373715	.18908568520356
.35	.45065120829171	.1545740393919
.40	.46300287320082	.11975275961889
.45	.47330865110306	.084940621696782
.50	.48266019956708	.050314757429
.51	.48449379002331	.043424498053437
.52	.48632594650336	.036546224164
.53	.48816889190114	.029686966462328
.54	.49001997676537	.022843887613529
.55	.4918859574488	.01601900202133
.56	.49377077632378	.0092141973979966
.57	.49567805823573	.002430802560521
.58	.49746498047765	$.3182641603775(10)^{-16}$

EIGEN VALUES FROM STABILITY ANALYSIS FOR

ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 4-b

$\beta = -.12$ $M_\infty = 1.0$

α	CREAL	CIM
0.0	.41167552342101	.1878
.05	.41623505215809	.18725085995722
.10	.42899625900601	.18183147831189
.15	.44724064021763	.17084403642609
.20	.46735879613637	.15384724143076
.25	.48699038772784	.13189101615667
.30	.50523940737027	.10660309868245
.35	.52236226626894	.079462427068201
.40	.53903028164075	.051725459886459
.45	.55586948135723	.024364982514944
.50	.57311212735258	$.1017881651868(10)^{-10}$

EIGEN VALUES FROM STABILITY ANALYSIS FOR

ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 4-c

$\beta = -.12$ $\qquad\qquad$ $M_\infty = 2.0$

$\underline{\alpha}$	CREAL	CIM
.00	.65839480434711	.026092947770033
.05	.65961725584558	.025915514056212
.10	.66326505733073	.02532970117879
.15	.66925926127822	.02417156096836
.20	.67741270635143	.022219596865938
.25	.68739697581636	.019268547734957
.30	.69877851277156	.015212623004071
.35	.71113049231316	.010050354226687
.40	.72400369336947	.0037646401254844
.45	.73962124009023	$.16030361443544(10)^{-7}$

EIGEN VALUES FROM STABILITY ANALYSIS FOR

ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 4-d

$\beta = -.12$ $\qquad\qquad$ $M_\infty = 3.0$

α	CREAL	CIM
.00	.799297248509	.01278
.05	.80000934947748	.01257
.10	.80229612802101	.01200
.15	.80664157054453	.01093
.20	.81359897034131	.00955
.25	.82704379652628	.0070382792707014
.30	.84460395081738	.0016267213425985
.35	.84622291302017	.0002287720441795

EIGEN VALUES FROM STABILITY ANALYSIS FOR

ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 4-e

$\beta = -.12$ $M_\infty = 4.0$

α	CREAL	CIM
.00	.86616538802484	.0106563485154l6
.05	.86682379170628	.01047
.10	.8690306186739	.00954
.15	.87538083224061	.007504028879073
.20	.88303325738425	.0055742297728253
.25	.88430275994354	.0010375977486407
.30	.87105365170367	$.14637401810687(10)^{-7}$

EIGEN VALUES FROM STABILTY ANALYSIS FOR

ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 5-a

$\beta = -.16$ $M_\infty = 0.0$

α	CREAL	CIM
0.0	.23720214020418	.26932
.05	.25150053447872	.26894
.10	.28707260466672	.26456999999997
.15	.32958872723365	.25198249969844
.20	.36871548175686	.23150484603023
.25	.40070454466847	.205651692272
.30	.42549834454723	.17685525583205
.35	.44445004587741	.14661692320698
.40	.45910305834033	.11572007798677
.45	.47080262511519	.08451680241725
.47	.47490565319875	.07198605088171
.50	.48063340969634	.05314223435821
.52	.48424269601754	.040555823420234
.55	.48946530220887	.021644245566366
.57	.49287533137931	.009017444924746
.60	.49714855115073	$.51051169643964(10)^{-17}$

EIGEN VALUES FROM STABILITY ANALYSIS FOR

ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 5-b

β = .-16 M_∞ = 1.0

α	CREAL	CIM
.00	.390132263	.17237509608007
.05	.3955289214209	.1715828304737
.10	.41073798666442	.16799041440433
.15	.43258525829471	.15921545104991
.20	.45661408648407	.14430445835654
.25	.47967243743361	.12435933593416
.30	.50047410317492	.10114810178348
.35	.51916572984406	.076171633290267
.40	.53645211078931	.050505479134
.45	.55310544219744	.024887069621686
.50	.56968563360666	$.17329650192198(10)^{-8}$

EIGEN VALUES FROM STABILITY ANALYSIS FOR

ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 5-c

$\beta = -.16$ $\qquad\qquad$ $M_\infty = 2.0$

α	CREAL	CIM
.00	.64373518113751	.028762839011559
.05	.64505248347879	.028669790868788
.10	.64899705474056	.028307094811578
.15	.65550184529697	.027456079565817
.20	.66435700576961	.025804622220761
.25	.67513011831739	.023068203047795
.30	.68719803701793	.019134401402838
.35	.69989839761269	.014121961288967
.40	.71269909323166	.0083015694599864
.45	.72552146348634	.0019185759632401
.50	.73729348133478	$.63750491393051(10)^{-8}$

EIGEN VALUES FROM STABILITY ANALYSIS FOR
ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 5-d

$\beta = -.16$ $M_\infty = 3.0$

α	CREAL	CIM
.00	.78589369980286	.015792655970023
.05	.78645640558174	.01568
.10	.78823521571603	.01556
.15	.79126704068021	.01509
.20	.79572124363865	.01429
.25	.80194239191989	.01307
.30	.81073262707369	.01095
.35	.82766576449979	.0061608409763784
.40	.8365888896946	.0017379727989417
.45	.83773192014701	.00022801991722174
.50	.83836888357047	$.11347246557158(10)^{-7}$
.20	.81163121710037	.0008406355497678
.25	.81170954271209	.001381226933045
.30	.81101006265947	.0026343082299221
.35	.80923721879293	.0017541354054970
.40	.8091763273152	.00079207227331372
.45	.8093510462839	.00031490868669327
.50	.80948673733201	.000069368519808528
.51	.79552652899238	.00010239920402293

EIGEN VALUES FROM STABILITY ANALYSIS FOR

ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 5-e

$\beta = -.16$ $M_\infty = 4.0$

α	CREAL	CIM
0.0	.85514992665401	.013524484229505
.05	.8556345141995	.0135
.10	.8570041638137	.01312
.15	.85948150102789	.01255
.20	.86369242071952	.01117
.25	.87184997281629	.00944
.30	.8897645753521	.0065286567249117
.35	.89259101440328	.0004986820466649
.40	.87954401262373	$.16539090895888(10)^{-7}$

EIGEN VALUES FROM STABILITY ANALYSIS FOR

ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 6-a

$\beta = -.19884$ $M_\infty = 0.0$

$\underline{\alpha}$	CREAL	CIM
0.0	.21990557686348	.17479832314286
.05	.23411447299982	.176992431878[illegible]
.10	.26964387451608	.1792
.15	.31285318997176	.17524942886964
.20	.35403327790082	.16390212269174
.25	.38926523792042	.14694742534449
.30	.41811265544226	.12650017547282
.35	.44144339834897	.10405249879606
.40	.46036741581478	.080480828726484
.42	.46693462193267	.070852927473832
.45	.4758926657407	.056259433700949
.47	.48134363506319	.046443551639306
.50	.48885307203288	.031613200452065
.52	.49347468826298	.021660449955077
.55	.49992351580529	.0066340660397154
.57	.50402011674148	$.27773507545248(10)^{-16}$

EIGEN VALUES FROM STABILITY ANALYSIS FOR

ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 6-b

β = -.19884 M_∞ = 1.0

α	CREAL	CIM
0.0	.37518469877919	.12698780076505
.05	.38138547962337	.12705
.10	.39899115583578	.1257679255834
.15	.42447921189644	.1203251835782
.20	.45286763600263	.10930370103203
.25	.48017931115488	.09362308948887
.30	.50469584422687	.075006498685015
.35	.52629068188418	.054959221348647
.40	.5454923691014	.034350507471669
.45	.56297179234488	.013666677486108
.50	.57934578533203	$.38928305239686(10)^{-17}$

EIGEN VALUES FROM STABILITY ANALYSIS FOR

ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 6-c

$\beta = -.9884$ $M_\infty = 2.0$

α	CREAL	CIM
.00	.62969487029807	.030889844737148
.05	.6311617302087	.030920641237614
.10	.63557624440788	.030936633398926
.15	.64291693941898	.030600424399213
.20	.65296076562186	.029432887092191
.25	.66512943144051	.026989425600505
.30	.67854728173942	.02309160066191
.35	.69232493104063	.017949937829179
.40	.70580292555574	.011984639078454
.45	.71860335851749	.0056504692929738
.50	.7307530289015	$.9193686072086(10)^{-8}$

EIGEN VALUES FROM STABILITY ANALYSIS FOR

ADIABATIC COMPRESSIBLE REVERSED FLOW BOUNDARY LAYERS

Table 6-d

$\beta = -.19884$ $M_\infty = 3.0$

α	CREAL	CIM
.00	.76882482147335	.02005818057211
.05	.76936714380275	.02008
.10	.77099503364300	.02008
.15	.77370202112201	.02008
.20	.77746621495485	.02000
.25	.78222893455565	.01962
.30	.78788619204293	.018864429978234
.35	.79428666460582	.017709024553168
.40	.80128832923942	.01607439528655
.45	.80875571423894	.013954395289655
.50	.81678746502399	.011351548861573
.55	.825557463186555	.0074586170736347
.60	.83481964201181	.0051050610659472
.65	.84925394836117	.003029645545813
.70	.86568773501342	.0089787871264246
.75	.85122932119987	.038191234101883
.70	.86060096659162	.034036401942572
.80	.84327106510608	.036561313531183
.85	.83664071110094	.030735203355245
.90	.83219222307689	.020560376004845

LIST OF SYMBOLS

$a^2 = \gamma RT$	square of the speed of sound
C_p	specific heat at constant pressure
C_v	specific heat at constant volume
$c = C_r + iC_i$	where C_r and C_i are real and $i = \sqrt{-1}$
C_r	propagation velocity
C_i	amplification factor
$e = C_v T + \frac{\bar{U}^2}{2}$	internal energy
$f'(\eta)$	defined by $\frac{df}{d\eta} = \frac{u}{u_e}$; dimensionless velocity ratio
$H = C_p T + \frac{1}{2}\bar{U}^2$	enthalpy
K	thermal conductivity
M	Mach number
$m = \frac{\beta}{2-\beta}$	pressure gradient parameter
$N = [\frac{m+1}{2} \frac{a_o}{\nu_o} \frac{M_e}{\xi}]^{1/2}$	y-stretching function
p	pressure
$\dot{q}$	heat transfer rate
R	gas constant
$S = \frac{H}{He} - 1$	enthalpy distribution
T	temperature
u	longitudinal velocity component
v	transverse velocity component
α	wave number
β	pressure gradient parameter
δ	boundary layer thickness
δ^*	displacement thickness

ξ	transformed similarity variable
η	transformed similarity variable
$\phi(y)$	small perturbation variable for transverse velocity
ψ	$1 + .2\, M_\infty^2$
ν	kinematic viscosity
μ	viscosity
γ	ratio of specific heats
$\underline{\underline{\tau}}$	stress tensor
ρ	density

SUBSCRIPTS

e	external flow
∞	far field flow
x	partial derivative with respect to x
y	partial derivative with respect to y
η	partial derivative with respect to η
$\hat{}$	small perturbation variable function of y
$\bar{}$	mean flow quantities

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